Theory of Linear Poroelasticity

with Applications to Geomechanics and Hydrogeology

Herbert F. Wang
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